OCR

RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4727

Further Pure Mathematics 3

Wednesday **25 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 3 printed pages and 1 blank page.**



**[Turn over**

1 Find the acute angle between the skew lines

$$\frac{x+3}{1} = \frac{y-2}{1} = \frac{z-4}{-1} \quad \text{and} \quad \frac{x-5}{2} = \frac{y-1}{-3} = \frac{z+3}{1}.$$ [4]

2 The tables shown below are the operation tables for two isomorphic groups $G$ and $H$.

| $G$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $d$ | $a$ | $b$ | $c$ |
| $b$ | $a$ | $b$ | $c$ | $d$ |
| $c$ | $b$ | $c$ | $d$ | $a$ |
| $d$ | $c$ | $d$ | $a$ | $b$ |

| $H$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| 2 | 4 | 8 | 2 | 6 |
| 4 | 8 | 6 | 4 | 2 |
| 6 | 2 | 4 | 6 | 8 |
| 8 | 6 | 2 | 8 | 4 |

(i) For each group, state the identity element and list the elements of any proper subgroups. [4]

(ii) Establish the isomorphism between $G$ and $H$ by showing which elements correspond. [3]

3 (i) By using the substitution $y^3 = z$, find the general solution of the differential equation

$$3y^2\frac{dy}{dx} + 2xy^3 = e^{-x^2},$$

giving $y$ in terms of $x$ in your answer. [6]

(ii) Describe the behaviour of $y$ as $x \to \infty$. [1]

4 (i) By expressing $\cos\theta$ and $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, or otherwise, show that

$$\cos^2\theta\sin^4\theta = \tfrac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2).$$ [6]

(ii) Hence find the exact value of

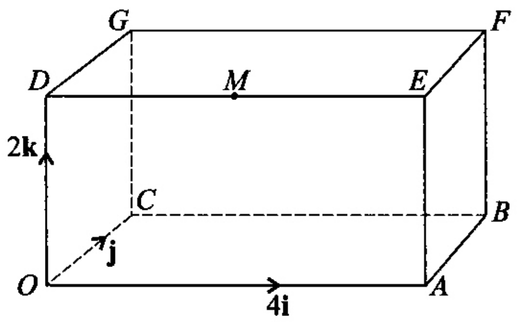
$$\int_0^{\frac{1}{3}\pi} \cos^2\theta\sin^4\theta \, d\theta.$$ [3]

5 (i) Solve the equation $z^4 = 64(\cos\pi + i\sin\pi)$, giving your answers in polar form. [2]

(ii) By writing your answers to part (i) in the form $x + iy$, find the four linear factors of $z^4 + 64$. [4]

(iii) Hence, or otherwise, express $z^4 + 64$ as the product of two real quadratic factors. [3]

6




The cuboid $OABCDEFG$ shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}$, $\overrightarrow{OC} = \mathbf{j}$, $\overrightarrow{OD} = 2\mathbf{k}$, and $M$ is the mid-point of $DE$.

**(i)** Find a vector perpendicular to $\overrightarrow{MB}$ and $\overrightarrow{OF}$. [3]

**(ii)** Find the cartesian equations of the planes $CMG$ and $OEG$. [5]

**(iii)** Find an equation of the line of intersection of the planes $CMG$ and $OEG$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [3]

7 A group $G$ has an element $a$ with order $n$, so that $a^n = e$, where $e$ is the identity. It is given that $x$ is any element of $G$ distinct from $a$ and $e$.

**(i)** Prove that the order of $x^{-1}ax$ is $n$, making it clear which group property is used at each stage of your proof. [6]

**(ii)** Express the inverse of $x^{-1}ax$ in terms of some or all of $x$, $x^{-1}$, $a$ and $a^{-1}$, showing sufficient working to justify your answer. [3]

**(iii)** It is now given that $a$ commutes with every element of $G$. Prove that $a^{-1}$ also commutes with every element. [2]

8 **(i)** Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4x = 0,$$

where $k$ is a real constant, in each of the following cases.

**(a)** $|k| > 2$

**(b)** $|k| < 2$

**(c)** $k = 2$

[8]

**(ii)** **(a)** In the case when $k = 1$, find the solution for which $x = 0$ and $\frac{dx}{dt} = 6$ when $t = 0$. [4]

**(b)** Describe what happens to $x$ as $t \to \infty$ in this case, justifying your answer. [2]